Seventh Vaidya-Raychaudhari Endowment Award Lecture

Subtle is the Gravity

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Abstract

In this lecture I build up the motivation for relativity and gravitation based on general principles and common sense considerations which should fall in the sphere of appreciation of a general reader. There is a novel way of looking at things and understanding them in a more direct physical terms which should be of interest to fellow relativists as well as physicists in general.

It is indeed a great honour and privelage for me to be asked to deliver the Seventh Lecture in the series which not only salutes the seminal and profound work of Professors Prahlad Chunilal Vaidya and Amal Kumar Raychaudhuri but more importantly also concretises love and affection they enjoy in the Indian relativity community. They symbolise the hardship and perseverance of a university academic and are truly the living academic “dharma”. I thank my colleagues on the Selection Committee for giving me this opportunity to pay my warmest tributes to them and share with you a viewpoint in understanding gravity.

With Professor Vaidya, I have dual (which is quite characteristic of gravity) relationship; on one hand we are “gurubhai” as students of Professor V. V. Narlikar and on the other I am his grand-student as a Masters student of Professor J. Krishnarao. I thank the latter for setting me on this trail. On this occasion I also remember Professor Narlikar with deep sense of reverence and gratitude, and I am greatly indebted to him for showing me the exciting and amusing way to look at things in one’s own way.

With Professor Raychaudhuri, it is different. There has been no such visible agents connecting us but somewhere deep down we seem to share much understanding and warmth. To him I have been a naughty student at a distance (which helps me to ask stupid questions) and that is how our dialogue goes on. After a long trying, I have recently succeeded in coauthoring a paper with him. That is something I would treasure.

Unlike some of my learned and renowned predecessors I do not ask for consideration and understanding for inadequacy of what I would say because there is no uncertainty and ambiguity about it. It has to be accepted as hard fact as such without much ado and pretension. This indulgence on my part stems from the belief that if you have seen a thing differently, then it is worth telling about it to others, never mind whether it is right or wrong. This is exactly what I intend to do in this lecture and I hope that it would not be too taxing for you.

1. At the very beginning

For the sake of completeness, let us begin at the very beginning with the Newtonian framework of classical physics. We have the three laws of motion, of which the first makes an equivalence statement that Uniform Motion (motion with a constant speed in a straight line) is equivalent to No Motion. That is it is impossible to distinguish between these two states by performing any physical experiment. Any deviation from this would signal presence or application of an external field or force. In a force free region, a particle would be in either of these states depending upon the initial condition. In here particle is an externally given entity, and no consideration is made about how does it come into existence, and what is the energy expended in creating it?

There is an overriding belief bordering to faith that nothing physically non-trivial happens without expense of energy and anything that has physical existence must have energy. That means the particle that enters in the Newtonian physics should also have energy even when it is at rest, let this be termed as its rest energy. With this bit of extrapolation, energy of a particle in free space would be the sum of kinetic energy and its rest energy. When particle is at rest, its entire energy is rest energy. How about a particle which is never at rest? That is, its entire energy is kinetic and rest energy is zero. Its existence is in its motion. Such a particle would be moving relative to everyone. With what speed, relative to whom, should it move? Since it must move relative to all, hence it must move with the same speed relative to all. If there exists such a particle in nature, there would exist a universal (invariant) speed which would be the limiting speed for all observers.

2. When there is light

Light is that thing which always moves and can never be at rest relative to anyone. It is an electromagnetic wave which propagates according to the Maxwell theory of electrodynamics. The speed of a wave is entirely determined by the properties of the medium in which it propagates. Then its speed can only be changed by moving the medium or changing it. If there exists a medium which is all pervading, it could neither be moved nor changed. Then the wave that propagates in such a medium would have a universal constant speed relative to everyone. Such a medium is called vacuum and light propagates in it which would hence have the constant invariant speed. A quantum of light is that particle, we envisaged earlier, with zero rest energy which cannot be at rest relative to any observer.

Thus there does exist in nature something physical that propagates with an invariant constant speed which is the limiting speed for all material particles. The incorporation of this fact would ask for radical modification of the Newtonian mechanics. The first obvious casualty would be the law of addition of velocities, because anything added to the velocity of light should give the same velocity of light. That is $w = u + v$ can no longer remain valid, it has to be changed. It should however remain the low velocities limit of the new law. Since the light velocity defines an upper threshold for propagation of any information, events would now be separated into two classes, the ones that are causally connected (whose time separation is greater than or equal to the time taken by light to travel from one event to the other) and the ones that are causally unconnected (whose time separation is less than the light travel time between them). Since there exists a universal velocity about which all observers agree, it would relate space and time and thereby knitting them together into a new entity called spacetime. We can measure distance in seconds, time taken by light to travel the distance, alternatively we can measure time in centimeters, distance traveled by light in the time.

In the spacetime diagram (it would be insulting for this audience to actually draw the diagram), the interior of the light cone indicates the causally connected region while the exterior causally unconnected. The distance between two locations would measure differently if we travel from one to the other by different paths;i.e. distance is a path dependent, as we know from everyday experience while traveling by an auto rickshaw/taxi. So must be the time between two events because of the equivalence of space and time implied by the universal constant speed of light. That is, the time between two events would dependent upon the path an observer takes to go from one event to the other. In principle, the time of the space voyage read off in an astronaut's clock would be different from his colleague's on the ground simply because they take different paths between the two events, the start and the end of the voyage. It is a different matter whether this difference is significant or not. Thus space and time measurements are relative observer dependent and it is the spacetime interval born out of marriage of the two has invariant observer independent meaning. "Space by itself and time by itself are to sink fully into shadows and only a kind of union of the two should yet preserve autonomy" thus spoke Minkowski in his famous Cologne lecture in 1908.

3. Light binds space and time

Newton's first law states that uniform motion is physically undetectable and thereby implying equivalence between frames that are in uniform motion for all physical phenomena. This is the principle of relativity. Added to this the fact of constancy of velocity of light gives rise to new mechanics which goes by the name, Special Relativity (SR). There is nothing more to it than simply deriving the logical consequences of these two basic facts. The salient features that emerge are: synthesis of space and time, path dependence of time and absence of absolute simultaneity for spatially separated events and the quadratic law of conservation of energy and momentum ($E^2 = p^2 + m^2$, where E, p and m are respectively energy, momentum and rest mass of the particle, and the velocity of light c has been set equal to 1) leading to the equivalence of energy and mass. The relation $E = mc^2$ has for lay audience become synonymous to Einstein and is often used to demonstrate the immense destructive power encoded in it. Note that conservation law is quadratic which distinguishes itself from the Newtonian linear law by the fact that a box containing two photons would have in general increase in its rest mass due to the presence of photons even though each photon has zero rest mass.

The frames referred above are the inertial frames (IFs) identified by the characteristic that free particles remain either at rest or in uniform motion. This is the behavior of free particles in absence of a force or field and it defines the standard state of motion. Any deviation from it is indicative of presence of a force or field. Force has to be applied externally by a mechanical agency while field is a non mechanical agency that mediates interaction between particles of specific (charge) property.

IFs are connected by the linear Lorentz transformation which keeps the velocity of light invariant ensuring its constancy for all IFs. As a matter of fact, this transformation is obtained by requiring invariance of the velocity of light for any two IFs. That means the universal character of velocity of light could be stated in a geometrical form as the invariance of spacetime interval defined as follows:

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2. \tag{1}$$

It is this spacetime (Minkowski) metric ("distance") which has the invariant observer (IF) independent meaning, and neither spatial distance by itself nor time interval by itself. Further it is the description of spacetime in absence of all forces, and free particles follow the geodesics (straight lines) of this metric. That is motion in absence of forces is characterized by geodesics of the Minkowski metric.

4. From flat to curved

One important point that emerges is that any universal physical property or statement must be expressible in a geometric form. As does the Minkowski metric for velocity of light and free particle motion in absence of force. This would serve as a good guiding principle as we go along.

To measure force we require an IF in actuality. The question is how to obtain it in reality? All forces must be removed. There are four basic forces in nature, of which the two are short range and hence they do not occur in macroscopic region. The other two are long range forces. Of which the electromagnetic force links only to particles having electric charge, and it could be shielded off by a suitable prescription for distribution of charges and currents. On the other hand the remaining force of gravitation has universal linkage and it therefore cannot be shielded off. Newton too did recognise the difficulty in removing gravity, and had prescribed to go infinitely away from matter distribution to actually realise an IF.

Since gravity cannot be removed globally, there cannot exist a global IF. However it can be removed locally as demonstrated by Galileo's famous experiment of the leaning tower of Pisa. All particles share the two properties, one inertia and other linkage to gravity (gravitational charge). The only one universal parameter available is mass (energy) for all particles. Thus the measure of inertia and gravitational charge must be the same, which would mean all particles irrespective of their mass, shape and substance would fall with the same acceleration under gravity. This was what for the first time experimentally tested by Galileo. In Einstein's words, gravity could be removed by letting oneself fall along with a freely falling lift. In contrast to Galileo, Einstein fortunately did this experiment only in thought and not in actuality, else he won't have been there to tell us the result and the following story of general relativity.

Freely falling lifts would thus define true IFs but only locally. That is, we can construct a LIF anywhere in spacetime but no GIF. The principle of relativity would now state that all LIFs are equivalent, and it is given the sophisticated name, Principle of Equivalence (PE). That is, by doing any physical experiment in actuality or in thought it is impossible to distinguish one LIF from the other. Conversely, gravity could be locally produced as well by giving upward acceleration equal to that of gravity to a lift in free space. This situation is indistinguishable from that of the inside of a lift at rest, say on the surface of earth. The strong form of PE states that all LIFs irrespective of their location in space and time would measure the same constant numerical values of the universal constants, the velocity of light c, the gravitational constant G and the Planck constant h.

The non existence of GIF has profound consequence that globally spacetime cannot be described by the Minkowski metric, which is flat having zero Riemann curvature. This clearly means that for honest and proper consideration of gravitational field, we will have to give up the comfort and ease of the non interacting inert flat spacetime typified by the Minkowski metric. This is a radical break. No other force interacts with background spacetime while gravity, it appears, cannot do without it. In the local neighbourhood we can always define a LIF, and write the flat Minkowski metric while globally the metric is non flat. That means spacetime must be curved in such a way that the effect of curvature is locally ignorable permitting existence of a LIF with the Minkowski metric. Clearly it must be a Riemannian manifold with the characteristic property of existence of a tangent plane defining a LIF. Note that the characterizing property for flat space is vanishing of the curvature rather than the form of the metric (the metric coefficients could be functions of coordinates yet the metric could be flat). This is to distinguish between globally removable inertial forces (centrifugal and coriolis) by coordinate transformation, and globally non removable force of gravitation.

It is the non removability of gravity which is responsible for curvature of the spacetime with non-Minkowskian metric given by

$$ds^2 = g_{ab}(x^i)dx^a dx^b. \tag{2}$$

Free particle motion would be given by the geodesic of this metric which would correspond to motion under gravity. Thus gravity is completely incorporated in the spacetime geometry and is described by curvature of the spacetime manifold. This leads to Einstein's theory of gravitation, called general relativity (GR). We have now to obtain the gravitational field equation from the curvature of spacetime.

5. Light bends space

In addition to the above strong motivation for curved spacetime for description of gravitation, here is yet another very strong and inescapable motivation. With the equivalence of mass and energy, energy in any form must do all that mass does. That means all that which has energy in any form must interact with gravitational field. How is particle's interaction with any field determined? By looking at the change in its velocity. Light however cannot by definition suffer any change in its velocity. Then how could it feel gravity? And that it must. In the Newtonian theory, a free particle instantaneously at rest experiences gradient of potential as acceleration which causes change in its velocity. Since photon (light) is never at rest relative to any frame, it cannot experience the gravitational force through gradient of potential.

This clearly brings forth the fact that if light has to interact with gravity, we will have to do something radically different. The pertinent consideration would then be what is it that light can respond to ?

Light propagates as a wave in space, hence it could only respond to changes in space. By light's interaction with gravity, what is expected is that it should bend like other particles rather than traveling along a straight line. This can alternatively be achieved by bending the space rather than light. The only sensible thing would then be that gravity must curve space and light simply propagates along the geodesic of the curved space. The physical consistency of all the principles involved thus demands that gravitational field must curve space. That is, a gravitating massive body curves space around it. With SR behind us, curvature of space should also imply curvature of spacetime. In fact, it is g_{tt} in the metric which generates $-\nabla\phi$ in the geodesic equation for ordinary particles. While for photon both g_{rr}

and g_{tt} contribute equally in the "space bending" (note that the Newtonian bending value was half of the correct value because it referred only to g_{tt}). It is the space that bends and not the light. Note however that the Weyl curvature of 4-spacetime must be non zero to make light feel the bending of space. What was for the first time measured by Eddington's team during the total solar eclipse of 1919 was not the bending of light but was in fact the bending of space. It is something like we see the sun going from east to west, but in fact it is the earth that is moving about the sun.

Gravitation could thus be described only through curvature of spacetime, and the theory that does this is GR.

6. Another view

Let us once again begin at the very basics. For existence of any physical entity which we would call by the general name particle, the minimal requirement is that it possesses some energy in some form. This is the bare situation. Such an entity could be characterized by a single parameter, the measure of its energy, which could be in the form of rest mass or purely kinetic like that of a photon. This characteristic would however be common for all other particles possessing additional characteristics like electric charge. This is thus a universal property of all particles.

All particles must interact with each other according to their characteristics. That means there must exist a universal interaction which is shared by all things. Since this is universal and would hence influence one and all irrespective of their location in space and time. The interaction must therefore be long range. By the term universal we would mean interaction with all forms of energy as well as at everywhere in spacetime. Such a universal field could not be removed globally so long as there exist some particles, in the limit at least one (at least two if you adhere to Mach's Principle), in the Universe. That is, it can never be completely and absolutely empty.

As argued earlier, because of the non removability of the universal interaction, there cannot exist a GIF and thus spacetime cannot be described by the Minkowski metric. Since the Minkowski metric provides a good background for the rest of physical phenomena, it must be available locally in a laboratory. That means there must exist LIFs in a local neighbourhood. In view of this, the metric should thus be written in such a way that it represents a curved Riemannian 4-spacetime manifold. The cause for curvature is the universal interaction which is shared by all particles. The universality demands that it must be synthesized in the spacetime geometry. It should then become simply a property of spacetime and is completely described by its curvature.

In any spacetime including the Newtonian space, the equation of motion of free particle is always free of mass of the particle. This is just to indicate that free particle motion is always determined by the geometry of spacetime/space. Free particle motion would thus be given by the geodesics of the curved spacetime metric. The geodesic equation is free of particle's mass. Since the interaction is universal, it cannot distinguish one particle from the other on the basis of mass. Another reason for absence of mass in the equation is that we also have particles with zero (rest) mass, and they are always free. This property is characteristic of the equation being determined by the spacetime/space as its geodesic.

We have thus a curved Riemannian spacetime of 4-dimensions which imbibes in its curvature the universal interaction. How do we deduce the equation of motion for the interaction? It is the curvature of spacetime which should yield the equation. The Riemann curvature satisfies the Bianchi differential identity, which on contraction yields the divergence free second rank symmetric tensor, called the Einstein tensor G_{ab}. The natural equation that could thus follow is:

$$G_{ab} = -\kappa T_{ab} + \Lambda g_{ab} \tag{3}$$

where κ and Λ are constants and T_{ab} is a symmetric tensor with vanishing divergence. This requirement ensures conservation of what the symmetric tensor T_{ab} may represent. The natural choice for that is the energy-momentum tensor of a matter distribution. The second term on the right is a constant for the covariant derivative relative to the curved metric g_{ab}.

The question is, could we make any sense out of this equation? At the outset, it looks a good equation ensuring the conservation of energy and momentum represented by the tensor T_{ab}, and Λ could be identified with the well-known cosmological constant. Let us solve it for $T_{ab} = \Lambda = 0$ for a radially symmetric metric. It is most remarkable that the solution includes in the first approximation gravitational field of a mass point in the Newtonian theory. This implies that the universal interaction is that of gravitation and the above equation is indeed Einstein's equation for gravitation in GR.

We have thus obtained Einstein's equation from a very general consideration which entirely hinges on universality. Isn't it amazing that it leads to a solution which agrees with the specific inverse square law of gravitation? Recall that the demand of existence of closed orbits for the central force does pick out the inverse square law of gravitation or electrostatics and the linear law of simple harmonic oscillator. Further demand of long range isolates the inverse square law.

How did this specific property get incorporated in our general consideration? Our main plank was universality which lead to curved spacetime and the above equation. The next step was to identify the energy momentum tensor of matter distribution with the tensor T_{ab}. This identification ensured conservation of energy and momentum. Of course it is well known that the Gauss law which implies the inverse square law is essentially a conservation law. It is perhaps the implied conservation law in our consideration that makes the bridge with the Newtonian gravity. At any rate, it is quite remarkable that the universality of interaction alone leads uniquely to Einstein's theory of gravitation.

From this standpoint, the view that spacetime should incorporate gravitation in its geometric structure is quite natural because of its universal character and it needs no further justification. Newtonian gravity is not quite universal (it was though universal enough for its times) as it does not include photons. We wish to say that GR results naturally and uniquely when the Newtonian theory is rendered truly universal. That is the complete universalization of the Newtonian gravitation is Einstein's theory of gravitation. This is the fundamental statement.

7. A quadratic force

The Lorentz force on a charged particle in electromagnetic field is a linear force law;i.e. it is linear in 4-velocity of the particle. Recently we have attempted to obtain the entire set of the Maxwell equations by demanding the force to be linear in 4-velocity in the relativistic law of motion. Apart from this we required the equation to follow from a Lagrangian, and there is no a priori choice between scalar and pseudo scalar charges. Then the solvability of the system ultimately leads to the Maxwell equations [1]. The question naturally arises, what would the quadratic law lead to? Perhaps to the Einstein equation, because gravitation is in a sense generalization of the electromagnetic field. That can only happen if the quadratic force requires the spacetime metric as its potential.

The relativistic law of motion is, $mdu^a/ds = f^a$, and we wish to consider the quadratic force, $f^a = T^a_{bc}u^b u^c$. The Lagrangian for a quadratic force would also have the quadratic term of the kind, $p_{ab}(x^i)u^a u^b$. This would on variation give the equation,

$$p_{ab}du^b/ds = -P_{(bc,a)}u^b u^c \tag{4}$$

where

$$P_{(bc,a)} = \frac{1}{2}(p_{ba,c} + p_{ac,b} - p_{bc,a}). \tag{5}$$

A comma denotes partial derivative.

The first point to note is that the equation is free of mass of the particle. This is the characteristic property of the quadratic force. This is an important inference which implies that the quadratic force would automatically obey PE. The equation when it is free of mass of the particle, it should be given by the geodesic of the spacetime metric.

The 4-force should be orthogonal to 4-velocity requiring $p_{ba,c}u^b u^c u^a = 0$ which is impossible as p_{ab} is symmetric. The equation of motion could make sense if and only if $p_{ab} = g_{ab}$ defines the spacetime metric. Then it would become the geodesic equation, indicating vanishing of the 4-acceleration relative to the metric g_{ab} and which would by definition be orthogonal to 4-velocity. The quadratic force thus requires spacetime metric as its potential which means it gets incorporated in spacetime geometry. This is the distinguishing and unique feature. The quadratic force comes through the Christoffel symbols of the metric, which are not tensors and involve first derivatives of the metric. The force could thus always be removed globally if space is flat but only locally if space is curved. There are two quadratic forces known to us; the inertial (centrifugal and coriolis) forces belong to the former category of global removability while gravitation belongs to the latter category of only local removability and of global non removability.

For the quadratic force to be universal that it is globally non removable, spacetime must be a curved Riemannian manifold. Once spacetime is curved, the Einstein equation could be obtained as done above in 3. The remarkable point is that the very general consideration of force being quadratic leads uniquely to the Einstein equation. Note that PE follows as a consequence of the quadratic law and not assumed a priori. The quadratic law thus characterizes mass proportional force fields and makes the spacetime metric dynamic when the force is globally non removable.

8. Some Subtelities

Newtonian theory was remarkably successful in explaining motion of planets in the solar system, and occurrence of eclipses and other such observable phenomena were predicted and verified at astonishingly high degree of precision. This raises a simple question what was then wrong with it which did not show up in these observations? The first and foremost in this context was interaction of light with gravity. This was not only difficult to observe but one did not even attempt to look for it as the theory did not predict it. It only became pertinent after the advent of SR.

8.1 *Gravitational Potential*

New theory always includes the old theory and one understands the former from the framework of the latter. It is therefore pertinent to consider the question, how does the new theory manifest in relation to the Newtonian theory? The Newtonian gravity is described by a scalar field satisfying the Laplace/Poisson equation, $\nabla^2\phi = 0, \rho$; where ρ denotes the matter density. The motion under gravity was described by the equation, $\ddot{\mathbf{x}} = -\nabla\phi$, which is free of mass of the particle by the prescription that gravitational force is mass proportional. The two main objections are: One, the former equation does not describe how the field propagates in space? Second, since gravitational field also has energy, which should also participate in gravitational interaction. That is the self interaction, gravity being its own source. This is something different and unique to gravity. Even in the empty space which is free of ponderable matter/energy, gravitational field energy would be present. Thus the correct equation for empty space should be $\nabla^2\phi = -\frac{1}{2}(\nabla\phi)^2$, a non linear differential equation.

To incorporate the particle equation of motion in the geodesic equation, it is sufficient to write $g_{tt} = 1 + 2\phi$. This is how the Newtonian potential enters into the metric. The gravitational field equation is as given by the equation (3). It is a second order non linear differential equations for the ten metric potentials which have four degrees of coordinate freedom implied by the Bianchi identities. The equation for empty space is obtained by setting $T_{ab} = 0 = \Lambda$, which is the analogue of the Laplace equation. It reads as $G_{ab} = R_{ab} - \frac{1}{2}R\, g_{ab} = 0$ which is equivalent to the Ricci tensor, $R_{ab} = 0$.

First of all let us enquire, do we solve the Laplace equation or the non linear equation containing the field energy density? Let us solve the vacuum equation for the field of a mass point. Consider the spherically symmetric

metric given by

$$ds^2 = Adt^2 - Bdr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2) \tag{6}$$

where A and B are functions of r and t. We are to solve the equation $R_{ab} = 0$.

It turns out that $R_{rt} = 0$ and $R^t_t = R^r_r$ would imply $A = 1/B = 1+2\phi(r)$, and then,

$$R^t_t = -\nabla^2\phi \tag{7}$$

and

$$R^\theta_\theta = R^\varphi_\varphi = -\frac{2}{r^2}(r\phi)' \tag{8}$$

where prime denotes derivative relative to r.

Note that we again solve the good old Laplace equation in $R^t_t = 0$ and not the one with the field energy density. Secondly, the other equation is its first integral which fixes the constant of integration to zero, determining the potential $\phi = -M/r$ absolutely, which can vanish only at infinity and nowhere else. And we obtain the asymptotically flat Schwarzschild solution. Note that asymptotic flatness was not due to the boundary condition but instead implied by the equations themselves.

Thus gravitational potential in GR is determined absolutely as constant potential would produce non-zero curvature, R^θ_θ, and hence would have physical meaning. This is something new in contrast to the classical physics where constant potential is physically ignorable because it had no physical effect.

Since we ultimately solve the Laplace equation, what has happened to the contribution of field energy? Though field energy must participate but it is a second order source and is not equivalent to the primary source of non-gravitational energy distribution. How could we distinguish between them? In the classical framework there exists no sensible way of doing it. Once again we are at the threshold of some new and novel construct. The only natural way is to curve the space so that contribution of the field energy is taken care of by its curvature leaving the Laplace equation intact. This is exactly what happens. If we consider space to be flat by setting $B = 1$, the equation $R^t_t = 0$ would have the non linear term of the field energy. It is the condition $A = 1/B$ which kicks out the non linear term and leaves only the Laplacian. Thus contribution of field energy goes into curving the space.

Since the gravitational potential $\phi = -M/r$ has very strong observational support, it couldn't have been tampered with. Here again space is

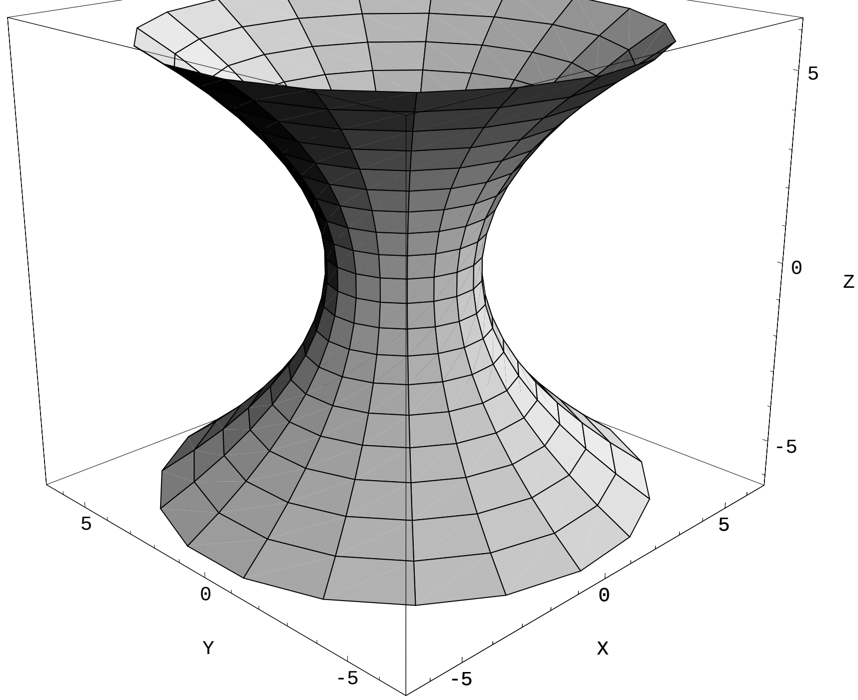

Figure 1: It shows the paraboloid of revolution for the parabola $z^2 = 8r - 16$, where $r^2 = x^2 + y^2$.

required to be curved to account for the field energy's contribution as was the case for negotiating light's interaction with gravitation. Both these are non-Newtonian effects. It is interesting to note that the non-Newtonian effects would primarily manifest through the space curvature.

8.2 *Negative Curvature*

There are theorems establishing positivity of non-gravitational energy, which produces the attractive gravitational field, whose energy would always be negative. The positive energy condition could alternatively be stated as the field energy being negative. This feature must be reflected in the space curvature which is caused by the field energy. The potential gradient pulls free particles towards the gravitating source, space curvature must also act in line with this motion.

Let us try to understand this phenomenon with the help of the Schwarzschild solution. For motion in the equatorial plane, the 2-space metric is given by

$$(1 - 2M/r)^{-1}dr^2 + r^2 d\varphi^2 \tag{9}$$

which has the negative curvature $-M/r^3$. It can be embedded into the 3-Euclidean space by writing $z^2 = 8M(r - 2M)$, which is a parabola and would generate a paraboloid of revolution (Fig.1). Clearly it has negative curvature which would tend free particles to roll down towards the centre and thus work in unison with the potential gradient [2].

Gravitational field energy thus produces negative curvature, and this would also be the norm for non-gravitational energy which is non-localizable. That is the proper energy condition for non-localizable energy distribution is that it be negative so as to work in unison with the localized energy. For instance, non-localized energy distribution for the charged black hole solution (the electric field energy) and also for the Schwarzschild - de Sitter solution is positive and hence it produces repulsive contribution in both cases opposing the attractive contribution due to mass.

This question has relevance in the context of a black hole on the brane [3], which would be sitting in a trace free energy distribution coming as a backreaction of the bulk on the brane. The question is what should be the sign of this energy density? It has been argued that it must be negative if it is to strengthen black hole's gravity.

8.3 *Empty Space*

In the Newtonian theory empty space is defined by vanishing of the matter density. Could we not define it similarly in GR at least for the simple case of space surrounding a static massive body? The analogue of the Newtonian density is the energy density measured by a static observer as defined by $\rho = T_{ab}u^a u^b, u^a u_a = 1$. In GR, we also have the null particles, and the energy density felt by them would be $\rho_n = T_{ab}k^a k^b, k^a k_a = 0$.

Shouldn't vanishing of these two densities be sufficient to characterize empty space for the situation under consideration? It turns out that it does [4]. The condition $\rho_n = 0$ for radially propagating null particles would imply $R^t_t = R^r_r$ which would lead to $AB = 1$. Putting this into the condition $\rho = 0$, equivalently $G^t_t = 0$ determines $A = 1 - 2M/r$, and so we obtain the Schwarzschild solution which characterizes empty space for spherical symmetry.

Thus for spherically symmetric spacetime, empty space could be defined in line with the Newtonian theory by the absence of energy density felt by both timelike and null particles. This is a physically interesting characterization.

8.4 *Black Hole*

Black hole is a body from which nothing can emanate out. There is a rigorous mathematical characterization involving sophisticated geometric constructs. Physically the most appealing definition is based on the consideration of escape velocity tending to the velocity of light. Over 150 years before the relativistic black hole was realised, Laplace had enquired when could light not escape a gravitating body? This was though simple argument and roughly pointing in the right direction. It is however an invalid (though valid in Laplace's time) application of a formula which happens to give the right result. The formula is invalid for $\frac{1}{2}m\,v^2$ is not the kinetic energy of a particle when its velocity tends to the velocity of light. Secondly, escape velocity being c does not prohibit things to emanate from any surface while black hole is a oneway surface, from which nothing can emerge.

I had been for long trying to devise a correct energetics argument for black hole. From what we have argued earlier, we could envision that gravitational charge, which could be defined in GR also for stationary field by appropriately adopting the Gauss theorem, is felt by the timelike particles through gravitational potential. On the other hand photons would feel the space curvature which is caused by the gravitational field energy. Photon is the limiting case of timelike particle. Now, what should happen when gravitational field energy becomes comparable to gravitational charge. Where this happens timelike particles should be tending to null particles. This is an energetics consideration characterizing black hole by the property of ordinary particles tending to photons.

The big question is how to get a measure of gravitational field energy which is notoriously ambiguous? There is a good prescription due to Brown and York [5] for total energy contained inside a sphere of radius r. By subtracting out mass at infinity from it we can get the measure of field energy which has the expected term, $-M^2/2r$ for large r. A black hole is characterized when gravitational charge coming from the Gauss theorem equals the field energy.

The Brown-York energy inside a sphere of radius r is given by

$$E(r) = r - (g_{rr})^{-1/2} \tag{10}$$

and then the field energy would be given by

$$E_F(r) = E(r) - E(\infty). \tag{11}$$

Our definition [6] for black hole is when

$$E_F = M_g = \frac{1}{4\pi}\int_{S^2} \mathbf{g}.\mathbf{ds} \tag{12}$$

where M_g is the gravitational charge and $\mathbf{g}$ is the red shifted proper acceleration as measured by an observer at infinity. It is the conserved mass M for the Schwarzschild black hole while for the charged black hole it reads as $M - Q^2/r$ where Q is the electric charge. It can be easily verified that our definition does give the right black hole radius for the spherical black holes. Further the characterizing relation has been rigorously established by using the Gauss-Codaci theorems and has also been applied to a black hole in the expanding universe [6].

We have thus given a physically illuminating and intuitively appealing characterization of black hole based on an energetic consideration. It is quite insightful. Note that the condition $g_{tt} = 0$ is the statement equivalent to escape velocity being equal to velocity of light (it defines the static limit, a particle has to move with velocity of light to remain static) while the condition $g^{rr} = 0$ defines the confinement of photons. Since photons are always free and cannot be given arbitrary velocity, their confinement inside a surface means that its curvature should be so strong that photons can't propagate out and the surface becomes oneway. This is exactly what the latter condition indicates.

8.5 *Electrogravity Duality*

It is the general property of a classical field to resolve it into its electric and magnetic parts. The manifestation of field due to a stationary source is called electric while that due to moving source magnetic. The terms electric and magnetic have been adopted because this resolution was for the first time realised for the electromagnetic field. One can transform from one to the other by a transformation which is given the name of duality transformation.

In the case of gravitation, the field is described by the Riemann curvature tensor which involves the second derivatives of the metric, and it thus has in fact dynamics of the field. In the case of electromagnetism, field contained the first derivative of the gauge potential, and dynamics was given by the field equation which is an independent statement involving second derivatives of the potential. Note that the field equation for gravitation is sort of implied by the Bianchi identity as shown above. We must therefore recognise the fact that gravitational field as described by the Riemann curvature is much subtler and higher order entity than electromagnetic field.

In the context of electromagnetic resolution, the Riemann curvature is double 2-form, while the electromagnetic field is described by a single 2-form. The projection of the 2-form and its dual onto a timelike unit vector u^a of an observer gives respectively the electric and magnetic part. A dual is defined by ${}^*F_{ab} = \frac{1}{2}\eta_{abcd}F^{cd}$ where F_{ab} is the electromagnetic field tensor and η_{abcd} is the 4-dimensional volume element. The Riemann tensor R_{abcd} has two 2-forms, the first two indices give the one and the last two the other. There would also occur two duals, left (on the first pair) dual and right (on the last pair) dual. To write the electric or magnetic parts each 2-form is to be projected on u^a; $E_{ab} = R_{acbd}u^c u^d$ is the electric part which is symmetric. The magnetic part would be given by $H_{ab} = {}^*R_{acbd}u^c u^d$ (projection of the right dual would give H_{ba}), which is trace free. Both of them would be orthogonal to u^a. E_{ab} will account for 6 Riemann components of the kind $R_{\alpha t\beta t}$ and H_{ab} will account for 8 components of the kind $R_{t\alpha\beta\gamma}$, and so there still remain 6 components unaccounted. These could be written as $\tilde{E}_{ab} = {}^*R^*_{acbd}u^c u^d$, which would denote the purely spatial components, $R_{\alpha\beta\gamma\delta}$. We would call $\tilde{E}_{ab}$ the passive electric part and E_{ab} as the active electric part.

Gravitation has two kinds of sources, active the non-gravitational energy distribution and passive the gravitational field energy which give rise respectively to active (E_{ab}) electric part and passive ($\tilde{E}_{ab}$) electric part. Note that we had argued earlier that the field energy produces spatial curvature which is denoted by the passive electric part. The magnetic part would be generated by motion of these sources. Note that electric parts are symmetric while magnetic part is not but is instead trace free. Thus any duality relation which does not impose any condition could only involve the electric parts. It turns out that if we consider the transformation involving all the three electromagnetic parts similar to the electrodynamics, the transformation would lead to the vacuum equation [7]. This has happened because the Riemann curvature does contain the dynamics of the field and the duality relations would prescribe certain relations between the curvature components giving the vacuum equation.

In electrodynamics, the duality transformation is the symmetry of the vacuum equation, which in this case is given by

$$E\,or\,\tilde{E} = 0, \;\; H_{ab} = H_{ba}, \;\; E_{ab} + \tilde{E}_{ab} = 0. \tag{13}$$

This would be invariant for the transformation,

$$E_{ab} \leftrightarrow \tilde{E}_{ab}, \ H_{ab} \rightarrow H_{ab} \tag{14}$$

which we call the electrogravity duality transformation [7]. The scalar curvature is given by $R = 2(E - \tilde{E})$, which would change sign under duality. And so would the Weyl curvature. The duality transformation would imply $GM \rightarrow -GM$ in the Schwarzschild solution. This happens because the duality means interchange of active and passive parts which are respectively anchored to positive non-gravitational matter/energy and to negative gravitational field energy.

In the familiar terms duality is equivalent to the interchange of the Ricci and Einstein tensors. This is because contraction of Riemann is the Ricci while that of its double dual is the Einstein tensor. The duality is the interchange between Riemann and its double dual. Thus at the level of equations, we just need to interchange Ricci and Einstein. The vacuum equation is however insensitive to this interchange and remains invariant. This is not so for non empty space. The duality transformation would for a perfect fluid imply $\rho \rightarrow \frac{1}{2}(\rho + 3p), p \rightarrow \frac{1}{2}(\rho - p)$. The de Sitter space is dual to anti de Sitter, while the radiation universe would be self-dual.

Though the vacuum equation is invariant under duality, the effective vacuum equation as considered in 8.3 above, $\rho = 0, \rho_n = 0$ is not. For spherical symmetry, the Schwarzschild solution is unique and hence all the equations admitting it as general solution would equally well characterize vacuum. Under the duality transformation, we have $\rho \rightarrow \rho_t, \rho_n \rightarrow \rho_n$ where $\rho_t = (T_{ab} - \frac{1}{2}Tg_{ab})u^a u^b$ is the timelike convergence density.

The dual equation to the effective vacuum equation would be $\rho_t = 0, \rho_n = 0$ which in terms of the familiar Ricci components would read as $R^t_t = R^r_r = 0$. It would solve to give $A = 1/B = 1 + 2\phi, \phi = -k - M/r$. That is it restores the additive constant in the potential as was the case for the Newtonian theory. The spacetime would be non empty and would have the stresses, $T^t_t = T^r_r = 2k/r^2$. In the Newtonian limit, it would not be distinguishable from the Schwarzschild solution. The dual solution thus has the basic features of the Schwarzschild solution but it is not asymptotically flat. To seat the Schwarzschild particle in a realistic situation, we have to break asymptotic flatness so as to allow for existence of other matter in the Universe without affecting the basic character of the field. This is precisely what has been achieved in the dual solution. The dual solution is thus Machian in spirit and is closer to the realistic setting.

This is also the Barriola - Vilenkin solution [8] describing asymptotically the field of a global monopole. Global monopole is a topological defect which is supposed to arise when global $O(3)$ symmetry is spontaneously broken into $U(1)$ in the phase transitions in the early Universe. Thus the duality transformation implies putting on a global monopole on the Schwarzschild particle.

Similarly the solutions dual to the NUT space, charged black hole and Kerr black hole have been considered. Further the applications of the duality to the dilaton gravity as well as in in higher and lower dimensions have been studied. In particular, an interesting new black hole spacetime has been obtained as a solution of the dual equation in 2+1 gravity [9]. It has also been shown that the solution of the static massless minimally coupled scalar field is also dual to the Schwarzschild field. This is obtained by writing ρ_n for, instead of radial, transversely propagating photons [10]. There could therefore occur more than one dual solutions, however they would all agree with the original solution in the first approximation.

9. End of everything

The breakdown of theory is called by the beautiful name of singularity. It indicates a situation where physical parameters of the theory become untenable by attaining infinite values. For GR, it is the spacetime curvature as well as density and pressure that should blow up at the singularity. The spacetime curvature becoming infinite indicates the breakdown of spacetime structure. The singularity in GR not only marks the end of GR, but also of everything else. No other physical theory or structure can survive in absence of the proper spacetime background. Thus like gravity its singularity is also universal.

There are powerful singularity theorems due to Penrose and Hawking that prove inevitability of occurrence of singularity in GR under quite general assumptions except one, the occurrence of compact trapped surfaces. A trapped surface essentially defines photon confinement inside a compact surface. The occurrence of trapped surface should rather be governed by the field equation. Taking this as an assumption is of course at non-trivial cost of generality which is the hallmark of the theorems. The influence of the theorems was however so strong and wide spread that the occurrence of singularity in GR is inevitable became a folklore. The observation of cosmic microwave background radiation implying the big-bang singular beginning of the Universe strengthened it immensely. This view was not correct was demonstrated by Senovilla in 1990 when he obtained a singularity free cosmological model filled with radiation with all physical and kinematic parameters remaining finite and regular everywhere [11]. The theorems became inapplicable because it did not satisfy the assumption of trapped surface. It was also a demonstration of the fact that violation of this assumption entails no unphysical features. The Senovilla class of models were cylindrically symmetric. Since then spherically symmetric imperfect fluid models having no singularity have been obtained, including the one which is also oscillating [12]. It oscillates between two regular states without ever becoming singular anywhere. That is there do exist truly non-singular cosmological solutions in GR. It is another matter whether they could be applied to the actual Universe.

One of the most important unresolved questions is, what is the ultimate end product of gravitational collapse? There is no question that it would end into a singularity as trapped surface would always form in this case. The question is, will it always be hidden behind a horizon (black hole) or sometimes visible (naked) to asymptotic observer? Penrose has proposed the cosmic censorship hypothesis to say that there exist no naked singularity in the Universe. The theorems only guarantee existence of singularity but give no clue about its nature and visibility to external world. There exists no proof or otherwise of the hypothesis. A considerable effort has been invested in studying the question in various settings (see a recent review [13]). It is perhaps now generally agreed that there exists regular initial data set which could lead to black hole as well as to naked singularity. It is being argued that cause for occurrence of naked singularity is inhomogeneity of density and presence of shear which make collapse incoherent and thereby delay formation of trapped surfaces. The race is between formation of singularity and trapped surface. If the latter forms first then it is black hole while if the former forms first then it would be visible thus naked to external observer before it is ultimately engulfed by the horizon. It appears that shear plays very important role in this process.

One of the ingenious applications of naked singularity is the proposal for the source of gamma ray bursts [14]. In practical terms we could model naked singularity as a seat of divergingly high curvatures. This could occur quite naturally in a massive collapsing object. The ultra high curvatures could produce a fireball giving rise to shocks in the on falling matter and ultimately result in producing high energy gamma rays. It happens for a very brief period before the trapped surface sets in and the object becomes a black hole. In this proposal gamma ray bursts could be envisioned as birth-cries of black holes. The crucial question is formation of singularity before setting in of the trapped surface which is delayed by presence of inhomogeneity and shear. Remarkably, this proposal is based purely on classical physics and invokes no new and speculative phenomena.

10. Looking ahead

Gravity is a universal interaction and it is therefore not a force like other forces but is a property of spacetime structure of the Universe. The question is how do we address the singularity in the structure of spacetime, indicating its breakdown not only for description of gravity but also as background for all physical phenomena. It indicates the end of everything, and should the answer of it be the Theory of Everything? So do the proponents of the string theory claim. The string theory is one of the proposals for a covering theory to GR which is vigorously being pursued. It proposes in principle a grand vision of synthesis of all forces, quantization of gravity and origin of matter in the Universe. All this is supposed to happen in 10 dimensions. The extra space dimensions are supposed to be compact and manifest only at a very high energies. GR would be the low energy limit of the theory. There is however no natural and unique way to come from 10 to 4 dimensions.

The other approach is that of canonical quantum gravity a la Abhay Ashtekar et al. How to tailor spacetime geometry so that it becomes amenable to quantization? It is essentially the quantization of spacetime geometry. New mathematical tools and constructs as well as new concepts are being developed so as to handle discretization of inherently continuum structure of spacetime. They are in fact evolving quantum geometry. This proposal remains confined to the familiar 4-dimensions.

Both the approaches have about equal share in success, for instance in understanding black hole entropy to some extent. Yet they have both long way to go. The string theory has a strong backup of the large particle physics community while the canonical qauntization is pursued mainly by a small group of relativists. The two approaches are quite different, the former relies on the field theoretic techniques and concepts while the latter on the geometric constructs. The two will have to converge as they asymptotically approach the ultimate theory of spacetime, which may or may not be a theory of everything! At any rate, one thing is certain that we do need a new theory which could be string theory or canonical quantum gravity, or something else!

Let us try to see into the unknown on the basis of wisdom gained from our past experience. From Newton to Einstein, the guiding force was light, its constant speed and its interaction with gravity. The latter required space to be curved. On the basis of these simple facts, we could argue that gravitational potential should not only give the acceleration to ordinary particles but also curve space for photons to feel gravity. Since acceleration is given by gradient, constant potential has no dynamical effect. But it is not so for space curvature, constant potential does produce non-zero curvature as shown above in 8.1. The journey from old to new theory is always of synthesis. For description of the field of a mass point, one cannot tamper with the Laplace equation of the Newtonian theory because of its strong observational support. At the same time, the potential must also curve the space which would require its absolute determination. That it can vanish only at infinity and nowhere else. Thus the GR in contrast to the Newtonian theory determines the potential absolutely. This is a synthesis.

Further, as there exists the escape velocity threshold for timelike particles, there should also exist a similar threshold for photons. Photons cannot turn back like ordinary particles, the only way they could be kept bound to a gravitating body is that they are not let to propagate out of a compact surface. That would happen when the surface becomes null;i.e. its normal is null. It then becomes a oneway membrane defining the horizon of a black hole. The existence of such a surface therefore becomes a natural requirement the moment we have photons to contain. These important and distinguishing features of GR could be deduced in principle without reference to the full theory.

The point I wish to make is that it should always be possible to anticipate and deduce some of the features of the new theory. Now when we wish to go beyond Einstein in the high energy regime, what kind of new features could one expect? Unfortunately, there does not seem to be anything like light showing us the way. One possible entity for synthesis is gravitational field energy like the gravitational potential in the Newton-Einstein synergy. In GR, it has no associated stress tensor and only comes through the space curvature. In the high energy limit, could it happen that it becomes concrete like other matter fields and attains a stress tensor? Something like this does happen in the string theory inspired brane world model [15]. In this model, all matter fields are supposed to remain confined to the 4-dimensional spacetime, called the brane while gravitational field can propagate in the 5-dimensional spacetime, called the bulk. The bulk is however taken as the vacuum with negative cosmological constant. Also is assumed the reflection symmetry in the extra space dimension. Then it turns out that it is not necessary to restrict the extra dimension to be finite and compact. By this construction, one of the remarkable things that happens is that the free gravitational field from the bulk gets "reflected" as backreaction onto the brane as trace free matter field. This is something in line with what one can extrapolate for a new synthesis. There is currently a good degree of activity in the brane world model.

The measure of a theory is the kind of questions it admits. New theory admits questions that were not admissible in the earlier theory. For instance, it is a valid question to ask in GR, when did the spacetime (the Universe) begin? And the answer is that it had its explosive birth in hot big-bang about 15-20 billion years ago. The next question in this series for the new theory to answer would be, what is the spacetime made of? That is the fundamental question which should be answered by the new theory.

A fundamental theory also has an impact on the world view. The view that has gained acceptance amongst people at large. For instance, the fact that the earth is not flat but is a curved surface like a ball has been internalized and assimilated in the knowledge base of the present day society. At a slightly higher degree of abstraction, the fact that material bodies attract each-other by an invisible force of gravity has also descended down to the common knowledge culture. The next step of advancement in the similar vein would be the assimilation of the fact that massive bodies curve the space around them. That is the abstract spacetime manifold (the Universe) we live in is not flat but is curved. One of the ways to measure this curvature is by measuring bending of light when it grazes a massive body like a star. Thus the geometry of spacetime should be of as general a concern as the geometry of the planet earth we live on. The only difference is that the former is an abstract entity while the latter is the concrete rock and sand.

Like gravity social interaction is also universal and of great consequence for our existence and well being. It should thus also attract collective concern and attention of all of us with utmost seriousness and commitment. We should never forget the fellow citizens who have been contributing from their honest earnings for our upkeep as well as for the facilities we use for our work. Apart from contributing to the knowledge base by our work in our specific discipline, as persons of learning and more importantly practitioners of scientific method we also owe to the society a studied and responsible participation in the discourses on the issues of wider social relevance.

Let us try to emulate the basic gravitational property of interaction with one and all, and come closer. If this is realised in the true spirit of both science and society, there is a good reason for hope for harmony, peace and an enlightened world community. This is a matter of gravity for one and all, once again a universal pronouncement of profound significance and value.

I close by quoting Ghalib, the great Urdu poet of the 19th century.

There are many poets in the world great and competent

yet they say different is Ghalib's manner.

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